True, False, and Open Sentences

CHAPTER 2

OVERVIEW

In this lesson, students first consider arithmetic sentences to decide if they are true or false. The lesson then introduces students to sentences that are neither true nor false but are algebraic equations, also called *open sentences*, such as $x + 3 = 7$ or $2 \times \square = 12$. Students learn to write equations using different variables and to figure out what numbers the variables represent in order to make the open sentences true.

BACKGROUND

Equations are mathematical sentences with equals signs. In an equation, the expressions on either side of the equals sign name the same quantity. Following are examples of arithmetic equations:

$3 + 4 = 7$

$6 \times 7 = 42$

$9 + 7 = 10 + 6$

$3 \times 8 = 32 - 8$

$\frac{2}{3} + \frac{1}{2} = \frac{7}{6}$

$10 - 6.5 = 3.5$

It's typical to talk about equations such as those listed as "math sentences" or "arithmetic sentences." This terminology is fine. However, it's beneficial to use "equation" interchangeably with "math sentence" and "arithmetic sentence" so that students become comfortable with this algebraic terminology as well.

In the elementary grades, students also have experience with sentences that use inequality signs, including $\neq$, $>$, $<$, $\geq$, and $\leq$. The following are true mathematical sentences:

$3 + 9 \neq 10$

$7 \times 4 > 8 \times 3$

$3 + 4 < 3 + 5$

Because they don't have an equals sign, these math sentences are *inequalities*, not equations. It's important to keep in mind that a math sentence must contain an equals sign in order to be called an equation. The focus of this chapter is on equations, not on inequalities.

Working with sentences in which the arithmetic is easy for students is helpful for introducing common mathematical conventions. For example, parentheses are useful punctuation to clarify equations, and this convention is introduced to students in the context of this lesson. In the following two sentences, the numbers and operations on the left side of the equals sign are the same, but the quantities on the right side are different:

$(5 \times 2) + 6 = 16$

$5 \times (2 + 6) = 40$

The parentheses used in math sentences indicate that you should perform the operations within the parentheses first. If no parentheses are included, however, then the mathematical convention called "order of operations" must be applied. This convention says to first perform all multiplication and division in order from left to right, and then to perform all addition and subtraction in order from left to right. Check the following equations written without parentheses to see how the order of operations convention has been applied:

$3 + 2 \times 5 = 13$

$2 + 6 \times 6 \div 2 = 20$

While parentheses aren't necessary in these equations for students who understand the convention for the order of operations, they are useful to include for clarity:

$3 + (2 \times 5) = 13$

$2 + (6 \times 6 \div 2) = 20$

But if the parentheses were placed differently in these equations, the quantities on the right side of the equals sign would change. For example:

$(3 + 2) \times 5 = 25$

$(2 + 6) \times 6 \div 2 = 24$

Another mathematical convention that's useful for students to learn is to use a dot to represent multiplication. For example, another way to write 5×3 is $5 \cdot 3$. Using the dot is especially helpful with algebraic equations to avoid confusing an x used as a variable with an $\times$ used to indicate multiplication. This convention is also presented in the lesson.

A difference between arithmetic equations and algebraic equations is that arithmetic equations are either true or false. Algebraic equations,

however, involve variables, and therefore aren't true or false. An equation such as $x + 3 = 5$, for example, becomes true if you replace the variable, x, with 2. Equations like this are also called open sentences, since they're open to a decision about whether they're true or false until you decide on the value for the variable. In this lesson, we use the term *open sentence* interchangeably with an *equation* with a variable. (Keep in mind that it's also correct to use *open sentence* to refer to algebraic inequalities, such as $\square + 3 < 5$ or $6 - x \geq 1$.)

In this lesson, we introduce various symbols for the variables in algebraic equations, but we mostly use boxes and triangles. These symbols are useful because you can write numbers in them and then do the arithmetic to see if the sentence is true or false. We also introduce the convention of using the same variable more than once in an open sentence, such as $\square + \square = 10$. When a variable appears more than once in an equation, it must represent the same number. Therefore, in $\square + \square = 10$, you must follow the convention and write the same number in both boxes. In order to make $\square + \square = 10$ true, you have to put 5 in both boxes. When an equation uses two different variables, however, such as $\square + \triangle = 10$, the variables can represent different numbers. In order to make $\square + \triangle = 10$ true, for example, you can put a 6 in the box and a 4 in the triangle, or a 2 in the box and an 8 in the triangle, or a 3 in the box and a 7 in the triangle, and so on. However, it's also OK for two different variables, such as the box and the triangle, to represent the same number; therefore, you can also make $\square + \triangle = 10$ true by writing 5 in both the box and the triangle.

All of these conventions are presented in the lesson in the context of the activities used to engage students. The emphasis is not on defining the conventions but on using them.

VOCABULARY

equation, open sentence, variable

MATERIALS

- 3"-by-3" sticky notes

TIME

- at least two class periods

The Lesson

Day 1

To begin class, I wrote the following mathematical sentences on the board:

$8 + 4 = 5 + 7$
$5 = 4 + 1$
$6 \cdot 0 = 6$

I read the first sentence aloud: "Eight plus four equals five plus seven." I then asked, "Is this true or false?" I paused just a moment and added, "Tell me in a whisper voice."

"True," the class whispered.

"Who would like to explain your thinking?" I continued. Most students put their hands up quickly. I called on Cameron.

"Eight plus four is twelve and so is five and seven," Cameron said. Several other students nodded their agreement.

"Does someone have a different idea?" I asked.

"Well," Dana said pensively, "another way of thinking about it is that if you take one away from the eight, then that is seven. If you put the one you took away from the eight with the four, then that's five. You can change eight plus four to seven plus five by just moving one from the eight to the four." No one had anything else to add.

"What do you think about the second equation?" I asked. "Is it true or false that five is equal to four plus one?"

"I think it's false," Marvin blurted out. "You can't put the answer at the beginning like that."

"What do the rest of you think?" I asked. I paused to give the students time to consider Marvin's point. It was early in the school year and I was still learning about my students' mathematical understanding. While Marvin's comment surprised me, it let me know that I needed to provide students with experiences involving different ways of writing mathematical sentences. A typical misconception that students have is to see the equals sign as an indicator that "the answer is coming" instead of as a symbol that indicates a relationship of equality.

"I think you can write it with the answer first," Rayna said.

"Please tell us more about your thinking," I encouraged.

"The equals sign means that both sides have to equal the same amount," Rayna explained. "Four plus one does equal five, so I think it's OK to write it the way it is on the board."

"I was thinking of it with candy bars," Jessie said. "You could have a group of five candy bars. You could split that group into a group with four and a group with one and it would still be five candy bars." Several students nodded as Jessie explained her idea. No one had anything else to add.

I then said, "It may look a bit strange to some of you, but it's mathematically OK to write an equation with the answer first. Is this second equation true or false?"

"True," the class responded.

"How do you read the third mathematical sentence?" I asked. Few students put their hands up. I suspected that the students were unfamiliar with using a dot to indicate multiplication. This convention is useful for students to learn and I've found it effective to introduce a new symbol like this one in the context of an activity when students are expected to make sense of and explain their reasoning.

"What does the dot mean?" I asked.

"It's a decimal point," Truc said.

"You could use it in money to separate dollars and cents," Carmen added.

"Those are both good ideas," I responded as I wrote *$1.25* on the board. "This is one way to write one dollar and twenty-five cents. What do you see that's different about the dot in one dollar and twenty-five cents and the dot in the third mathematical sentence?"

"In money, the dot is called a decimal point," Joshua said.

"That's correct," I responded. "What else do you notice?"

"The decimal point is lower in money than in the third problem," Jazmin said. "In the third problem the dot's in the middle and there's space between the dot and the numbers, so it looks like two numbers instead of one number like with money."

I responded, "Yes, Jazmin, the dot is in the middle. When you write a dot like that, it isn't a decimal point. It means multiplication. You can use a dot in this way instead of the times sign that you usually use for multiplication."

"I know about the third problem now," Tawny said. "You read it 'Six times zero equals six,' and that's false."

"I agree," Diego added. "Zero times anything is zero because zero groups of anything is zero, or a million groups of zero is still zero."

"At first I thought the dot was the middle of the plus sign and it meant plus," Tony said. "If it did mean plus, then it would be true because six plus zero is six."

"Show me using your thumb if you think the third mathematical sentence is true or false," I said. "If you think it is true put your thumb up, if you think it's false put your thumb down, and if you're not sure put your thumb sideways." Most students put their thumbs down. I didn't worry about the students who were unsure or who responded incorrectly because in a moment, all of the students would have the chance to think of sentences that were true and false. Then I'd be able to check on students who hadn't been sure about the last sentence.

A Class Assignment I then said, "With your partner, take the next few minutes to think of at least one example of an equation that's true and at least one example of one that's false. Write them on a sheet of paper so you don't forget them, and then some of you will have the chance to share your ideas." I circulated, listening to students and observing what they were writing. When most pairs had at least one true and one false mathematical sentence, I asked for their attention.

"In just a moment I'm going to give you a chance to share your ideas," I said. "I'm going to make two columns on the board, one for true mathematical sentences and a second for false mathematical sentences." As I explained this to the students, I drew two columns on the board, labeling one *True* and the other *False*. "When I call on you, just read us your mathematical sentence. Don't tell if you think it is true or false. We'll guess and see if you agree with our guess." I called on Tony and Lucy.

"Three times four," Tony said.

"That's only part of a mathematical sentence," I said. "You need to complete it."

"Oh, three times four equals twelve," Tony corrected himself.

"Thumbs up if you think Tony and Lucy's sentence is true, thumbs down if you think it is false, and put your thumb sideways if you aren't sure," I said. All thumbs were up. "Do you agree, Tony and Lucy?" They nodded. I wrote their equation in the True column and asked, "Who has another?"

"Ours is sort of tricky," Rayna began. "You multiply six times three and divide that by two. Then comes the equals sign. On the other side you do four plus five."

I paused to give students time to think. I noticed some seemed confused,

so I asked Rayna to repeat herself. There was a mix of responses, with some students indicating their agreement, others their disagreement, and some not sure.

"It looks to me like some students think it's true, some think it's false, and a few aren't sure," I said. "Let's see if we can figure it out together. Rayna, please come up and write your mathematical sentence up to the equals sign."

Rayna came to the board and wrote:

$6 \times 3 \div 2$

After they had a few moments to think about what Rayna wrote, several students raised a hand. "Please call on someone to figure out how much this is worth," I said. Rayna called on Terry.

"It's nine," Terry said. "Six times three is eighteen, and eighteen divided by two is nine."

"I agree!" Rayna said.

"What's the rest of your mathematical sentence?" I asked.

"Equals four plus five," Rayna replied. She completed her sentence on the board:

$6 \times 3 \div 2 = 4 + 5$

"Show me using your fingers how much four plus five is," I instructed the class. The students quickly put up nine fingers. Rayna nodded her head, indicating that she agreed. "Both sides of the equation are worth nine. Is Rayna's mathematical sentence true or false?" I asked.

"True," the class chorused. I added her equation to the list under True.

"Who has a mathematical sentence or equation that you think is false?" I asked. I called on Jazmin.

"Six plus one is equal to six minus one," Jazmin said.

"False," the class responded.

"It has to be false because six is the same on both sides and if you add one on one side and take away one on the other, automatically it has to be false," Carmen said with conviction. I recorded Jazmin's equation on the chart under False.

"Who else would like to share?" I asked. I called on Tawny.

"One times one equals one plus one," Tawny said.

"Show me with your thumb if you think it is true, false, or you're not sure," I said. Thumbs went down.

"Why do you think it's false?" I asked.

"Easy!" Pablo said. "One times one is one. One plus one is two. One doesn't equal two." No one else had anything to add. I added Tawny's sentence to the chart under False.

True	False
$3 \times 4 = 12$	$6 + 1 = 6 - 1$
$6 \times 3 \div 2 = 4 + 5$	$1 \times 1 = 1 + 1$

We processed several more equations in the same way. Kiko offered $12 \cdot 4 = 24 + 24$, Cameron shared $15 \div 3 = 3$, and then Joshua shared $4 + 1 = 10 - 4$. For each, students decided if they were true or false and explained their reasoning.

True	False
$3 \times 4 = 12$	$6 + 1 = 6 - 1$
$6 \times 3 \div 2 = 5 + 4$	$1 \times 1 = 1 + 1$
$12 \cdot 4 = 24 + 24$	$15 \div 3 = 3$
	$4 + 1 = 10 - 4$

Introducing Open Sentences I knew that more of the students wanted to share their equations, but I wanted to move on with the lesson. Also, I wanted to give the students who hadn't had a chance to share an opportunity to do so. I said, "I know many of you still have equations you'd like to share. Later, I'll give you each a sticky note and you can write an equation on it and post it on the chart in the appropriate column." Then, as the students watched, I wrote the following on the board:

$5 + \square = 13$

"Is this equation true or false?" I asked. The class was quiet. Finally a few hands went up. I waited a moment more to give the students a chance to think, and then I called on Jazmin.

"It could be either," Jazmin said. "We don't know what the box is, so we don't know if it's true or false."

"That's what I think, too," Garrett said.

"How could we make it true?" I asked.

"Change the box to eight," Jeremy suggested.

"Or just write eight in the box because five and eight equals thirteen," Lizzie said.

"I'll write eight in the box," I said. "Your idea is fine, Jeremy, but if I write an eight in the box, then we'll still have a record of the original open sentence." I wrote *8* in the box:

$5 + \boxed{8} = 13$

"Is there any other number I could write in the box that will make the sentence true?" I asked.

"I don't think so," Truc said thoughtfully. "All the other numbers make it wrong. Like, seven is wrong and so is nine. I think the only way to make it true is to put eight in the box." Several other students nodded as they listened to Truc explain his thinking.

Elyssa had a different idea. "You could write 'four plus four' in the box and that would make it true," she said. "Four plus four is the same as eight."

"Yes," I responded, "four plus four is another name for eight, and there are many different ways we can write eight. But is there anything that isn't worth eight that we can write in the box to make the open sentence true?"

"I think you could make it work by fractions," Chase said. "You could put sixteen over two as the answer and that would make it true." As I wrote $\frac{16}{2}$ on the board, several hands went up.

"Sixteen over two looks different, but it's really the same amount," Jessie said.

"It's still eight," Tina added.

I replied, "The equation would be true as long as whatever we put in the box is equivalent to eight." No one had any further comments.

"Mathematical sentences like this one are called *open sentences*," I said. "They're neither true nor false because there's a part of the sentence—the box in my equation—that isn't a number. The box is called a *variable*, because you can vary what number you put into it or use to replace it." I wrote *open sentence* and *variable* on the board. I planned to use this vocabulary regularly to help students become familiar and comfortable with it, just as periodically throughout the lesson I used the word *equation* rather than *mathematical sentence*.

"Would 'seven times six equals box' be an open sentence?" Jazmin asked. I wrote on the board:

$$7 \cdot 6 = \square$$

"Does someone have a thought they'd like to share about Jazmin's question?" I responded as some students raised their hands. Allowing students to respond to one another in this kind of situation helps build confidence and can increase understanding. I called on Rayna.

She explained, "I think it's an open sentence because you can't tell whether the mathematical sentence is true or false. It depends on what goes in the box. It could be true, but you could also make it be false.

Lucy added, "Forty-two should go in the box if you want the problem to be true. If you put thirty-nine in the box instead, then it's false."

"There are lots of ways to make it false," Tony said.

"What about 'four plus box equals twelve'?" Joshua asked. I wrote on the board:

$$4 + \square = 12$$

"Joshua wants to know if this is an open sentence," I said. "Put your thumb up if you think it is, put your thumb down if you think it isn't, and put your thumb sideways if you're not sure." All thumbs went up immediately.

"I agree that it's an open sentence," I said. "Why is it?"

"I know," Turner said, clearly excited. "Because whether or not it's true or false depends on what goes in the box."

"What would make it true?" I asked. "Show me with your fingers." The students put up eight fingers.

"What would make it false?" I asked.

"Anything would make it false except for what makes it true!" Terry said. The class giggled. "Well, anything would make it false except for eight, so all other numbers make it false."

When I've taught this in other classes, students don't always suggest open sentences, as Jazmin and Joshua did. In those situations, I write open sentences on the board and ask the students questions as I did for the open sentences that Jazmin and Joshua offered.

I then said to the class, "Work with your partner for a few minutes to come up with some other open sentences." A class discussion allows only some of the students to contribute. Having students work in pairs gives all of them a chance to take an active role. After a few minutes, I noticed that all of the students had written at least three open sentences, and I asked for the class's attention. I called on Lizzie to share first.

"How about 'fifteen thousand plus one equals box'?" Lizzie said.

I wrote on the board:

$15{,}000 + 1 = \square$

"Use your thumb to show me what you think," I said. "If you agree that Lizzie's equation is an open sentence put your thumb up, if you disagree put your thumb down, if you aren't sure put your thumb sideways." All thumbs were up, showing agreement with Lizzie.

"Who has an idea about what number to write in the box to make Lizzie's open sentence true?" I asked. Practically all of the hands went up. I called on Diego.

"Fifteen thousand and one," he said.

"Who would like to come up to the board and write fifteen thousand one?" I asked. Fewer hands were raised now. Some children weren't sure that they could write the number correctly. I called on Keith.

"But it won't fit in the box," he said.

"I can make the box bigger," I responded. I did so and Keith came to the board and correctly wrote *15,001*.

Introducing Other Variables I then called on Kenny to give another open sentence. "Triangle minus four equals three," Kenny said.

I wrote on the board:

$\triangle - 4 = 3$

"Show me what you think with your thumbs," I said. The students indicated with their thumbs that they thought Kenny's mathematical sentence was an open sentence.

"Who knows what number to put into the triangle to make the open sentence true?" I asked.

"Seven," Dana said. The others agreed.

It's important for students to learn that we can use different symbols for variables. I was pleased that Kenny had volunteered the use of a triangle. If no student had, however, I would have written an open sentence as Kenny did, talked about it with the students, and then introduced other symbols as well. Since Kenny made his suggestion, I built on it at this time. Underneath Kenny's equation, I wrote:

$\square - 4 = 3$

I said, "I think that my open sentence is the same as Kenny's in one way and different in another way. Who thinks they know what I'm thinking?" Hands shot up.

"You used a box instead of a triangle," Tawny said.

"Yes, I used a box for the variable and Kenny used a triangle for the variable," I said, taking the opportunity to use the word *variable*.

"But the numbers are the same," Terry said. I then wrote on the board:

$x - 4 = 3$

"What about this equation?" I asked. The class was quiet so I asked, "Is it an open sentence?" Some thought it was and others weren't sure.

"Who would like to use your own words to explain what an open sentence is?" Several hands went up. I called on Tony.

"It's a sentence that has a box or something that stands for a number,"

Tony explained. "It depends on what number you put in whether or not it's true."

"Is there something anyone else would like to add to what Tony said?" I asked. No one had another idea to share.

"So what do you think about the sentence I wrote with the x instead of a box or a triangle? If you think it's an open sentence, show thumbs up; show thumbs down if you think it isn't an open sentence; and thumbs sideways if you're not sure." Three students showed their thumbs sideways and the rest showed thumbs up.

"Who would like to explain why you think it's an open sentence?" I said. I called on Terry.

He said, "It's like Tony said—it has something that stands for a number that's missing. I think you can use whatever you want. An x is OK, so is a box, so is a triangle."

"I agree with Terry," I said.

"Can you use any letter?" Lucy asked.

"Yes," I replied. "Actually, you could use any symbol you'd like. But mostly we see boxes, triangles, and letters used for variables in equations." I then asked, "Who else would like to give an open sentence?" I called on Dana.

"I have one with fractions," she said. " 'Box plus one-half is equal to one.' I know what to put into the box. It has to be one-half because one-half plus one-half is one."

I wrote on the board:

$$\square + \tfrac{1}{2} = 1$$

"I agree that putting one-half in the box makes the open sentence true," I said. "Is Dana's equation an open sentence?" Thumbs were up.

Introducing Other Algebraic Conventions Before class was over, I wanted to introduce to the students the idea that when a variable is used more than once in an equation, it represents the same number. I wrote on the board:

$$\square + \square = 10$$

"Who notices how this open sentence is different from the others we've been talking about?" I asked.

The class was silent for a moment. Then Steve's hand flew up and, at the same time, he blurted out, "There are two boxes!" A murmur rippled through the classroom as the others acknowledged Steve's discovery.

"Can you do that?" Lucy wanted to know. "I mean, is it still an open sentence?"

"Yes," I replied. "It's fine to have more than one box in a sentence, or more than one triangle or more than one x, or even some of each."

"There are lots of numbers that work," Terry said.

"Tell me more about what you're thinking," I said.

"It could be ten plus zero, or six plus four, or five plus five, like that," he explained.

"I agree that there are different combinations that add to ten," I said, "but there's an important rule for equations like this one that we have to follow. When a variable appears more than once in an open sentence, then it must represent the same number everywhere it's used in that problem.

That means that in the open sentence I just wrote, both boxes have to stand for the same number. Your arithmetic figuring was correct, Terry, but in this open sentence, whatever I put in one box I must put in the other. How can we make this open sentence true and follow this rule?"

"So it has to be five?" Terry said.

I wrote *5* in each box, saying as I did so, "If I put a five in this box, then I have to put it in the other box, too. So now the equation says, 'Five plus five equals ten,' and that's true."

$\boxed{5} + \boxed{5} = 10$

"What number could I write in the boxes to make this equation false?" I asked. Hands went up and I called on Scott.

"Six, or seven, or any other number will make it false," he said.

I then wrote on the board:

$\square \times \square = 16$

"How can I make this open sentence false and also follow the rule that the box stands for the same number each time it appears?" I asked. Hands flew up.

"Put fives in the boxes," Pablo said. As I wrote *5* in the boxes, Pablo added, "Five times five is twenty-five, not sixteen."

"What about making the open sentence true?" I asked.

"Put four in the boxes," Rayna said. I erased the *5*s and wrote *4* in the boxes. The class agreed that the sentence was now true and followed the rule.

I had time for one more sentence. To introduce the idea of using two different variables in the same equation, I wrote on the board:

$\square + \triangle = 10$

"Hey, can you have a box and a triangle in the same sentence?" Kenny asked.

"Yes, you can," I answered. "Talk with your neighbor about how to make this sentence true."

This typically causes a little confusion. Because the variables are different, you don't have to put the same numbers in them. Therefore, there are several different ways to make the equation true, as Terry had suggested earlier when I wrote $\square + \square = 10$ on the board. But it's also possible for the different variables to represent the same numbers, so it's also correct to write *5* in both the box and the triangle. I wanted to explain this convention to the students. I called them back to attention and called on Carmen.

She said, "We think there are all the different ways to add numbers to ten, like Terry said before, but you can't do five and five because the box and triangle are different."

"You're almost completely right," I responded to Carmen. "It's OK to put different numbers in the box and triangle because they are different symbols, but it's also OK to put five in both the box and the triangle. If the variables are the same, the numbers they represent have to be the same. If the variables are different, they can be anything, which includes also being the same number. This is another bit of information about algebra that's important to remember."

"So five and five is OK?" Jazmin asked.

"Yes," I said.

It was now the end of the period. I gave an assignment. "For homework, make up five open sentences and figure out how to make them true."

"Do we have to use boxes?" Dana asked.

"You can use boxes, triangles, or any letter, as long as each of your open sentences has at least one variable," I responded. I then wrote an example on the board to model for the students how to record:

$$(2 \times \square) + 7 = 15$$

"Is this an open sentence?" I asked the class.

Lucy explained, "Yes; you can't tell if it's true or false because it has a box."

"I put the parentheses in so that you would be sure to multiply the two by the box first before adding on the seven," I said. It isn't necessary to use parentheses since the convention is always to multiply first before adding, but it's OK to include them for clarity.

"What number do I have to put into the box to make the open sentence true?" I asked. "Raise your hand when you've figured this out." I waited until most hands were up. I then asked the students to say the number in a whisper voice. Most students said, "Four," but I heard a few other numbers, too.

"Let's test to be sure that four makes the sentence true," I said and wrote *4* in the box.

Carmen explained, "Two times four is eight, and eight plus seven is fifteen. So four is right."

"Yes, four makes the open sentence true," I said. I wrote on the board:

$$(2 \times \square) + 7 = 15 \qquad \square = 4$$

"Can we make them hard?" Tony asked.

I said, "As long as you can figure out how to make your equations true, you can make them as hard as you like. My goal is to be sure that you know how to write an open sentence. I'll be checking that each of your mathematical sentences is complete with an equals sign and that each contains at least one variable."

At the end of class, I distributed sticky notes to students who wanted to record their true or false sentences. I planned to start each math class over the next few days with analyzing whether the equations posted were true or false.

Day 2

At the beginning of class, after talking about three of the equations on sticky notes and categorizing them as true or false, I gave the students directions about what to do with their homework from the night before. "Please share your paper with the person sitting next to you. When you look at your partner's paper, your job is to check that he or she has written five open sentences. You don't have to figure out what makes them true; just verify that they are complete open sentences. If you have any questions, talk with your partner. If you both can't resolve your issue, then raise your hands."

I gave the students a few minutes to check each other's papers. A few pairs called me over to ask about putting in parentheses, and a few found arithmetic errors, but their discussions went smoothly. Figures 2–1 through 2–3 show three students' homework.

FIGURE 2–1 Steve liked using large numbers in his open sentences but had incorrect solutions for equations three and five.

1. $\square + 74 = 145$ $\qquad \square = 71$
2. $(11 \times \square) + 6 - 17 = 44$ $\qquad \square = 5$
3. $(\square \times 102) + 15 - 7 = 118$ $\qquad \square = 5$
4. $(15 \times \square) + 10 - 7 = 183$ $\qquad \square = 12$
5. $(8 \times \square) + 4 - 8 = 80$ $\qquad \square = 8$

FIGURE 2–2 Justin helped Matt add parentheses to his sixth equation, but neither boy noticed the error in the first equation.

Equations

1. $\square \times 11 + 5 = 71$ $\qquad \square = 66$
2. $\square \times 5 - 10 = 20$ $\qquad \square = 6$
3. $11 + 6 + 10 = \square$ $\qquad \square = 27$
4. $2 \times 4 + \square = 16$ $\qquad \square = 8$
5. $9 \times 5 + 9 = \square$ $\qquad \square = 54$
6. $(\square + 15 + 5) \times 6 = 150$ $\qquad \square = 5$

FIGURE 2–3 Tessa used multiplication, addition, and subtraction in each of her equations.

1. $\square \times 10 - 30 + 5 = 15$ $\square = 4$
2. $\square \times 6 + 4 - 10 + 5 = 35$ $\square = 6$
3. $\square \times 12 + 2 - 3 = 107$ $\square = 9$
4. $\square \times 7 - 3 + 15 = 75$ $\square = 9$
5. $8 \times 6 - \square + 10 = 52$ $\square = 6$

Bonus: $\square \times 2 + 5 = 11$ $\square = 3$

I then called the class to order and said, "Each of you should choose one of your open sentences to present to the class. You'll tell it to me, I'll write it on the board, and then the rest of the class will try to figure out what makes it true." Cameron went first. I recorded her equation on the board:

$72 + \square = 100$

After giving the class a chance to think, I called on Chase. "I think it's twenty-eight," he said. Cameron nodded.

"How did you figure that out?" I asked Chase.

Chase explained clearly, "First I thought that seventy-two plus eight made eighty, and then I knew I needed twenty more to get to one hundred. So eight and twenty is twenty-eight, so seventy-two plus twenty-eight makes one hundred." I recorded Chase's thinking on the board:

$72 + 8 = 80$
$80 + 20 = 100$
$8 + 20 = 28$
$72 + 28 = 100$

I wrote Chase's answer on the board next to Cameron's open sentence:

$72 + \square = 100 \qquad \square = 28$

Next Lizzie gave her open sentence and I recorded it on the board:

$(\square \times 5) + 3 - 20 = 8$

"Try to figure out what number to write in the box to make Lizzie's open sentence true," I said. I gave the students a few moments to think. Most were trying different numbers, and this guess-and-check approach gave the students useful practice with mental computation. I called on Pablo.

"Write five in the box," he said, and then explained how he calculated. "Five times five is twenty-five, plus three is twenty-eight, and twenty-eight minus twenty is eight."

We continued for the rest of the period with students presenting their open sentences for others to solve. Several times, a student presented an equation for which he or she had an incorrect solution. In those instances, I asked the student to rethink the work by either revising the equation or the number he or she had identified to make it true. For example, Steve's third equation, $(\square \times 102) + 15 - 7 = 118$, stumped the class. When Steve revealed his answer of 5, others disagreed and convinced Steve that 5 didn't make the equation true. I asked the class to help Steve correct his work and gave them several minutes to work in pairs. When I called the class to attention, I gave Steve the first chance to report. He had revised his equation so that 1 was the correct answer: $(\square \times 102) + 15 - 7 = 110$. Lucy, however, had another suggestion. She said, "Change the multiplication sign to an addition sign and then 8 works." We tried Lucy's suggestion and found that for her revised equation, $\square + 102 + 15 - 7 = 118$, the number 8 made it true.

After about half of the students had had a turn, Nick asked, "Can we show more than one?"

I answered, "Right now, I'm going to be sure that everyone who wants to has a chance to present one open sentence. Then we'll see if there's time for repeats."

"Do we have to use the ones on our sheets or can we think of other ones?" Carmen asked. She hadn't yet presented an equation.

"You can think of other ones if you'd like," I answered.

Over the next several days, every student had a chance to present at least one open sentence.